# Distributed Framework for Branch-and-Price

Master's Dissertation Proposal

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## Abstract

This dissertation proposes a distributed framework for solving complex branch-and-price optimization problems, leveraging parallelism and the Message Passing Interface (MPI) protocol. Branch-and-price algorithms, which combine branch-and-bound with column generation, are essential for solving large-scale integer programming problems but pose significant computational challenges. The research aims to develop innovative algorithms and a distributed computing framework to enhance the efficiency and scalability of branch-and-price methods. These algorithms will be implemented in modern programming languages and applied to real-world optimization problems. Comparative analyses with state-of-the-art optimization solvers will assess the performance of the proposed distributed framework. The findings will be disseminated through reports, publications, and presentations, contributing to advancements in parallel optimization and practical applications.

**Key-words:** Branch-and-price, distributed computing, MPI protocol, parallel optimization, integer programming.

## 1) Introduction

Optimization problems across various domains often involve large-scale integer programming, which can be effectively tackled using branch-and-price algorithms. These algorithms combine the branch-and-bound technique with column generation to handle the exponential growth of decision variables. However, the computational demands of branch-and-price algorithms are substantial, requiring innovative strategies to enhance performance and scalability (Barnhart et al., 1998).

In transportation and logistics, branch-and-price is commonly used for solving vehicle routing problems (VRP) and crew scheduling problems (Desaulniers et al., 2005). These applications involve numerous constraints and large datasets, necessitating efficient computational techniques. Figure 1 illustrates the basic structure of a branch-and-price algorithm.

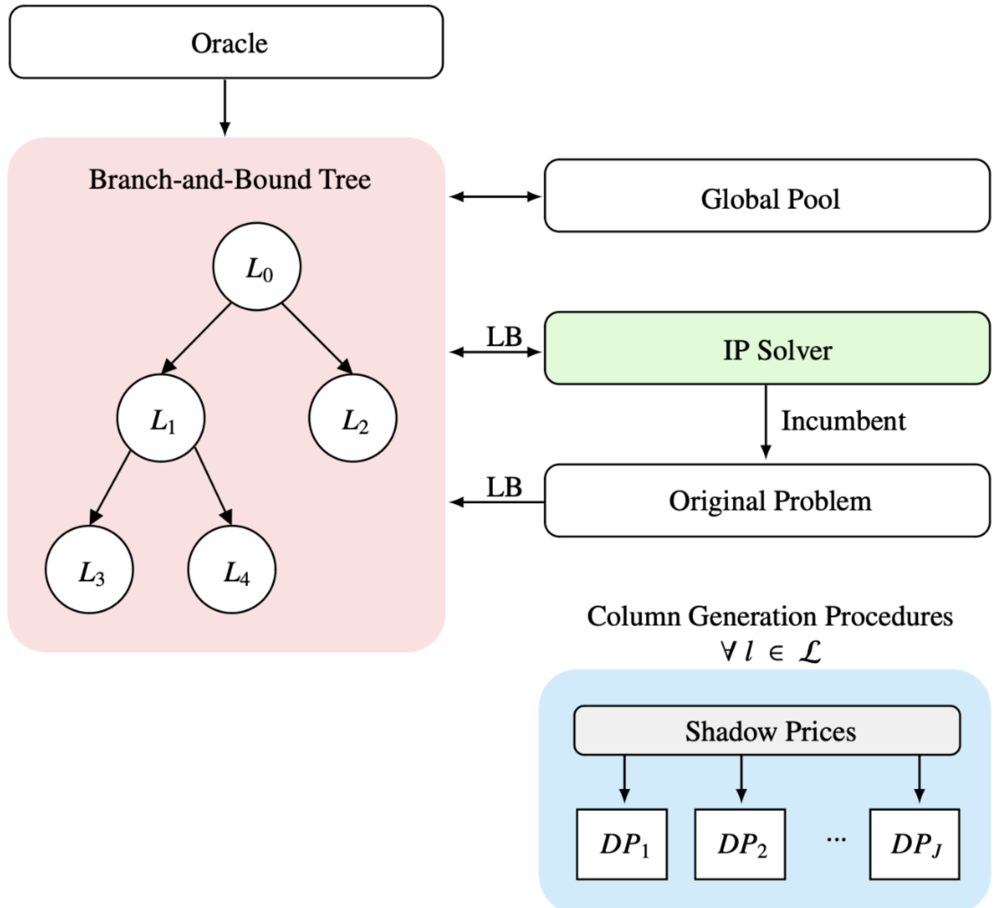


**Figure 1.** Example of a branch-and-price algorithm flowchart.

The Message Passing Interface (MPI) protocol offers a robust framework for developing distributed applications. By enabling parallel processing, MPI can significantly reduce computation times for complex optimization problems. Recent research has demonstrated the effectiveness of branch-and-price algorithms in various applications, from airline scheduling to telecommunications network design (Pecin et al., 2017), and can be further enhanced by parallelization techniques.

This dissertation aims to harness the power of MPI and parallel computing to develop a distributed framework for branch-and-price algorithms. By distributing the computational workload across multiple processors, the framework seeks to improve the efficiency and scalability of branch-and-price methods, enabling the solution of larger and more complex problems.

## 2) Objectives

This dissertation aims to advance the state-of-the-art in distributed optimization algorithms for branch-and-price, focusing on the integration of MPI protocol and parallel computing techniques. The specific objectives are:

- Design a distributed framework for branch-and-price algorithms that leverages MPI for parallel computation.
- Implement the distributed framework in a modern programming language, such as C++ or Python, to facilitate integration with existing optimization libraries.
- Apply the distributed framework to representative optimization problems, including VRP, crew scheduling, and telecommunications network design.
- Compare the performance of the distributed framework with state-of-the-art optimization solvers, such as Gurobi and CPLEX.

## 3) Related Works

Ralphs, Ladányi, and Trotter (2003) provide an extensive review of branch, cut, and price (BCP) algorithms, which extend the branch-and-price framework by incorporating cutting planes. Their work on SYMPHONY, a generic framework for implementing BCP, addresses the challenges associated with dynamically managing sets of cuts and variables throughout the search tree. SYMPHONY's modular design facilitates its application across various problem settings and hardware platforms, highlighting the benefits of parallelism in BCP algorithms. The insights from their research underscore the potential of parallel computing to solve very large-scale discrete optimization problems effectively.

Testa and Notarstefano (2022) explore the application of distributed branch-and-price algorithms in multi-robot systems. They propose a purely distributed approach where each agent locally solves small linear programs, generates columns by solving simple knapsack problems, and communicates a fixed number of basic columns to its neighbors. Their work demonstrates finite-time convergence to an optimal solution, applying the algorithm to a generalized assignment scenario for a team of robots. The implementation in a Robot Operating System testbed underscores the practical applicability of their distributed branch-and-price approach.

## 4) Methodology

The research will follow a systematic approach to design, implement, and evaluate the distributed framework for branch-and-price. The steps are outlined as follows:

1. **Literature Review (Months 1-6):** Conduct a comprehensive review of existing literature on branch-and-price algorithms, MPI protocol, parallel computing techniques, and relevant application domains. Identify challenges and opportunities for improving the efficiency and scalability of branch-and-price methods through distributed computing.
2. **Algorithm Design (Months 5-10):** Design a distributed framework for branch-and-price algorithms, incorporating MPI for parallel computation. Explore innovative strategies for

task distribution, load balancing, and communication management to optimize performance.

3. **Implementation (Months 6-13):** Implement the designed framework in a suitable programming language, such as C++ or Python, ensuring compatibility with existing optimization libraries. Develop modular and well-documented code to facilitate integration and testing.
4. **Application (Months 11-15):** Apply the developed framework to representative optimization problems, such as VRP, crew scheduling, and telecommunications network design. Conduct rigorous testing to validate the correctness and efficiency of the implemented algorithms.
5. **Performance Evaluation (Months 13-16):** Conduct comprehensive performance evaluations of the proposed framework, comparing the solution quality and computational time with state-of-the-art solvers (e.g., Gurobi, CPLEX). Evaluate scalability and robustness across different problem sizes and settings.
6. **Documentation and Reporting (Months 14-18):** Document the findings in technical reports and prepare papers for submission to academic journals and conferences. Disseminate results to both academic and practitioner communities to promote the practical application of the developed framework.

## 4) Expected Results

The expected contributions resulting from this dissertation are:

- A novel distributed framework for solving branch-and-price problems using MPI.
- Implementation of the distributed framework in modern programming languages for easy integration.
- Application of the framework to real-world optimization problems in transportation, logistics, and telecommunications.
- Comparative analysis of the framework's performance against state-of-the-art solvers.
- Dissemination of findings through reports, publications, and presentations.

## References

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